Problem 3: If one earthquake is 31 times as intense as another, how much larger is its magnitude on the Richter scale?

Answer: 1.49

Solution: $I_1 = 31\ I_2$

We are looking for the quantity M_1 -- M_2

$$M_1 = \log \frac{I_1}{S}$$

$$M_2 = \log \frac{I_2}{S}$$

$$I_1 = 31\ I_2$$

$$M_1 - M_2 = \log \frac{I_1}{S} - \log \frac{I_2}{S}$$

$$M_1 - M_2 = \left(\log \frac{I_1}{S}\right) - \left(\log \frac{I_2}{S}\right)$$

$$M_1 - M_2 = (\log I_1 - \log S) - (\log I_2 - \log S)$$

$$M_1 - M_2 = (\log 31 I_2 - \log S) - (\log I_2 - \log S)$$

$$M_1 - M_2 = (\log 31 + \log I_2 - \log S) - (\log I_2 - \log S)$$

$$M_1 - M_2 = \log 31 + \log I_2 - \log S - \log I_2 + \log S$$

$$M_1 - M_2 = \log 31$$

$$M_1 - M_2 \approx 1.49$$

The larger earthquake was 1.49 larger on the Richter scale.

Let's check our answer.

$$M_1 - M_2 = 1.49 = \log \frac{I_1}{S} - \log \frac{I_2}{S}$$

$$1.49 = \left(\log \frac{I_1}{S}\right) - \left(\log \frac{I_2}{S}\right)$$

$$1.49 = (\log I_1 - \log S) - (\log I_2 - \log S)$$

$$1.49 = \log I_1 - \log I_2$$

$$
\begin{aligned}
1.49 &= \log \frac{I_1}{I_2} \\
10^{1.49} &= \frac{I_1}{I_2} \\
\frac{I_1}{I_2} &\approx 30.9029543251 \\
\frac{I_1}{I_2} &\approx 31 \\
I_1 &\approx 31 I_2
\end{aligned}
$$

The check will not be exact because we rounded the answer. However, it is close enough for checking.